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PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1843. No. 39.

February 27.

SIR Wm. R. HAMILTON, LL.D., President, in the Chair.

Sir William Betham exhibited to the Academy an ancient shoe, found in a bog on the property of Sir Nicholas Fitzsimon, in the King's County.

Professor Lloyd read a supplement to a former paper "on the Determination of the Intensity of the Earth's Magnetic Force, in Absolute Measure."

In a paper recently communicated to the Academy, the author had shown that the ratio of the coefficients of the first two terms, in the expression for the moment of the force exerted by a deflecting upon a suspended magnet, was generally given by the formula

$$h = 2\frac{\text{M}_3}{\text{M}} - 3\frac{\text{M}'_3}{\text{M}'};$$

in which M and M_3 denote certain integrals depending on the law of distribution of free magnetism in the deflecting magnet, and M' and M'_3 the corresponding quantities for the suspended magnet. It was further shown, that when the magnets were small, this formula was reduced to

$$h = \tfrac{3}{5}(2l^2 - 3l'^2);$$

l and l' denoting the half lengths of the two magnets. This ratio being thus known *à priori*, the two unknown quantities in the equation of equilibrium of the suspended bar are reduced to one; and we are thus enabled not only to dispense with the observations of deflection at two separate distances, but also to infer the quantity sought with much greater accuracy than in the received method, by superseding the process of elimination.

The preceding value of the ratio, h, has been derived from an approximate law of magnetic distribution, which can be regarded as physically exact only in the case of very small magnets; and the truth of the formula has been verified, in that case, by direct experiment. It was interesting to inquire, therefore, how far the same formula represented the law of action of large magnets, and whether, by any modification, it might be applied to the results obtained with such instruments.

For this purpose the following deflection experiments were made:—The magnets employed were rectangular bars, 12 inches, 9 inches, and $7\frac{1}{2}$ inches, in length; $\frac{7}{8}$ of an inch in breadth; and $\frac{1}{4}$ of an inch in thickness. The observations were made with the aid of the *Unifilar Magnetometer* of the Magnetic Observatory, which has been elsewhere described; and simultaneous observations were taken with the *Declinometer*, in order to eliminate the changes of declination which occurred in the interval of the opposite deflections. In the first and second series, the position of the suspended bar was observed by means of a collimator attached; in all the rest, it was observed by the help of a mirror connected with the stirrup, which reflected the divisions of a scale placed at a distance of nearly six feet.

The angles of deflection were calculated, in the case of the collimator bar, by the formula

$$\tan u = \tfrac{1}{2}(n_e - n_w)k;$$

where n_e and n_w denote the observed readings of the scale, with the marked end of the deflecting bar to the East and to the West respectively. The value of the constant k is given by the formula

$$k = \left(1 + \frac{\mathrm{H}}{\mathrm{F}}\right) \tan\theta ;$$

θ denoting the angle corresponding to one division of the scale, and $\frac{\mathrm{H}}{\mathrm{F}}$ the ratio of the torsion force to the magnetic force. When the mirror is employed, the formulæ of reduction are similar, if only we substitute $2u$ and 2θ for u and θ. The following Table gives the values of the constants employed in the reduction:

l	θ	$\frac{\mathrm{H}}{\mathrm{F}}$	$\log k$
·5 (coll.)	49″·158	·00180	6·37795
·5 (mirr.)	58″·766	·00162	6·75643
·375 —	. . . . .	·00255	6·75684
·313 —	. . . . .	·00292	6·75700

The following Tables contain the calculated results: the values of $\frac{1}{2}(n_e - n_w)$ are corrected for the changes of declination which occurred in the interval of the two readings n_e and n_w.

SERIES I. and II. $l = l' = {\cdot}5$.

D.	I.		II.	
	$\frac{1}{2}(n_e - n_w)$	u	$\frac{1}{2}(n_e - n_w)$	u
4.5006	194·78	2° 39′ 45″	194·22	2° 39′ 18″
6.0010	82·51	1° 7′ 43″	82·26	1° 7′ 30″·5

SERIES III. and IV. $l = {\cdot}5$, $l' = {\cdot}375$.

D.	III.		IV.	
	$\frac{1}{2}(n_e - n_w)$	u	$\frac{1}{2}(n_e - n_w)$	u
4·0010	242·30	3° 56′ 25″	242·52	3° 56′ 38″
5·0015	123·31	2° 0′ 53″	123·50	2° 1′ 4″
6·0019	71·40	1° 10′ 4″·5	71·38	1° 10′ 3″

SERIES V. and VI. $l = ·5$, $l' = ·313$.

D.	V.		VI.	
	$\frac{1}{2}(n_e - n_w)$	u	$\frac{1}{2}(n_e - n_w)$	u
4·0005	243·58	3°57′ 44″·5	242·74	3°56′ 56″
5·0008	123·65	2° 1′ 15″·5	123·36	2° 0′ 58″·5
6·0010	71·41	1°10′ 6″·5	71·25	1° 9′ 57″

SERIES VII. and VIII. $l = ·375$, $l' = ·5$.

D.	VII.		VIII.	
	$\frac{1}{2}(n_e - n_w)$	u	$\frac{1}{2}(n_e - n_w)$	u
3·3762	211·18	3°26′ 10″·5	210·50	3°25′ 30″·5
4·3754	97·88	1°35′ 55″	97·56	1°35′ 36″·5
5·3757	53·24	0°52′ 13″	52·88	0°51′ 51″·5

SERIES IX. and X. $l = l' = ·375$.

D.	IX.		X.	
	$\frac{1}{2}(n_e - nw)$	u	$\frac{1}{2}(n_e - n_w)$	u
3·3766	212·37	3°27′ 31″	212·52	3°27′ 40″
4·37 60	97·52	1°35′ 39″·5	97·55	1°35′ 41″·5

SERIES XI. and XII. $l = ·375$, $l' = ·313$.

D.	XI.		XII.	
	$\frac{1}{2}(ne - n_w)$	u	$\frac{1}{2}(n_e - n_w)$	u
3·3766	214·16	3°29′ 21″	213·83	3°29′ 1″
4·3760	98·05	1°36′ 13″	97·90	1°36′ 3″·5
5·3765	52·85	0°51′ 54″	52·82	0°51′ 52″

The following Table contains the calculated results of the foregoing observations. The values of the coefficients, Q and Q′, are deduced from the formula

$$\tan u = \frac{Q}{D^3} + \frac{Q'}{D^5}$$

by the method of least squares.

l	l'	Q	Q′	$\frac{Q'}{Q} = h.$
·5	·5	4·2800 4·2662	– 0·824 – 0·790	– 0·193 – 085
·5	·375	4·3934 4·3976	+ 0·293 + 0·293	+ 0·067 + 0·067
·5	·313	4·3791 4·3760	+ 0·887 + 0·692	+ 0·202 + 0·158
·375	·5	2·3830 2·3702	– 0·823 – 0·761	– 0·345 – 0·321
·375	·375	2·3407 2·3402	– 0·161 – 0·135	– 0·069 – 0·058
·375	.313	2·3456 2·3428	+ 0·027 + 0·005	+ 0·012 + 0·002

The values of h thus obtained are not adequately represented by the formula which has been already deduced for the case of small magnets, the differences between the calculated values and the means of the two observed results being, in general, greater than the differences of the latter *inter se*. It was accordingly natural to inquire whether the agreement might not be rendered more complete by pushing farther the approximation in the value of the function which represents the law of magnetic distribution. This was found to be the case on trial. But it was also found that the observed results were represented, with nearly equal exactness, by the empirical formula,

$$h = 2(l - c)^2 - 3(l' - c)^2;$$

a formula which agrees with the hypothesis, that the whole force of each magnet is concentred in two points, or poles, at given equal distances from the ends. If we expand the preceding formula, and add together the resulting equations, we have for the determination of c,

$$6c^2 + 2\Sigma(2l-3l')c + \Sigma h - \Sigma(2l^2-3l'^2) = 0;$$

or, substituting the numerical values of the coefficients deduced from the preceding Table,

$$c^2 - 0{\cdot}625\,c + 0{\cdot}0425 = 0;$$

from which we deduce $c = {\cdot}078$.

The following Table contains the values of h thus calculated, together with the means of the observed results. The differences barely exceed the probable errors of the latter; and the corresponding error in the calculated value of Q is less than the probable error of the same quantity, as deduced in the ordinary method from the observed deflections at two distances.

l	l'	h (obs.)	h (calc.)	Diff.
·5	·5	− 0·189	− 0·178	+ ·011
·5	·375	+ 0·067	+ 0·092	+ ·025
·5	·313	+ 0·180	+ 0·191	+ ·011
·375	·5	− 0·333	− 0·358	− ·025
·375	·375	− 0·063	− 0·088	− ·025
·375	·313	+ 0·007	+ 0·011	+ ·004

It follows from the preceding formula, that the relation between the half lengths of the two magnets, which will cause the coefficient of the fifth power of the distance to vanish, is

$$l - c = 1{\cdot}224\,(l' - c);$$

or, substituting for c its value,

$$l + {\cdot}0175 = 1{\cdot}224\,l'.$$

It will appear evidently from the foregoing results, that on account of the large probable error of h, its value should be determined in each case from the mean of a much greater number of observations, before we can obtain thereby a satisfactory verification of any formula for its calculation. As far as the comparison has been here carried, the results appear to indicate that the value of h cannot be obtained *à priori*, in the case of large magnets, with that precision which would justify us in superseding observation, although we may obtain thereby an approximate value, comparable in exactness with the result of a single observation.